Problem Sheet 6

1. (4 points) Let K be an algebraically closed field of characteristic zero. Determine an algebraic closure of $K((T))$ and determine the absolute Galois group.

2. (4 points) Does the field $\mathbb{Q}((T))$ possess a Galois extension with Galois group S_6? If yes, give an example, or a disproof. Can such an extension be totally ramified?

3. (2+1+1 points) (a) Suppose L/K is a separable extension of prime degree n. Show that for all $\gamma \in \mathfrak{m}_L$

$$N_{L/K}(1+\gamma) = 1 + N_{L/K}(\gamma) + \mathrm{Tr}_{L/K}(\gamma) + \mathrm{Tr}_{L/K}(\delta)$$

for some $\delta \in \mathcal{O}_L$ with $v_L(\delta) \geq 2v_L(\gamma)$.

Suppose now that L/K is unramified of arbitrary degree n.

   (b) Show that $\lambda_{0,K} N_{L/K}(x) = N_{k_L/k_K} \lambda_{0,L}(x)$ for all $x \in U_L$.

   (c) Show that $\lambda_{i,K} N_{L/K}(x) = \mathrm{Tr}_{k_L/k_K} \lambda_{i,L}(x)$ for $i \geq 1$.

   where $\lambda_{i,K}$ denotes the isomorphisms between $U_K^{(i)}/U_K^{(i+1)}$ and the additive resp. multiplicative group of the residue field for K (and analogously $\lambda_{i,L}$ for L).

4. (2+2 points) *(Newton polygons continued)* Let $f = 1 + a_1 T + a_2 T^2 + \ldots$ be a power series with Newton polygon $N(f)$.

   (a) Let $b = \sup\{\lambda\colon \lambda \text{ is a slope of } N(f)\}$. Prove that the convergence radius of f is p^b. *(Assume first that f is not a polynomial. Then find an interpretation such that the statement also makes sense for polynomials.)*

   (b) Find f such that $N(f)$ has an irrational slope.

Please hand in your solutions in the lecture on Tuesday, 20th of November. You may work in groups of at most three students.